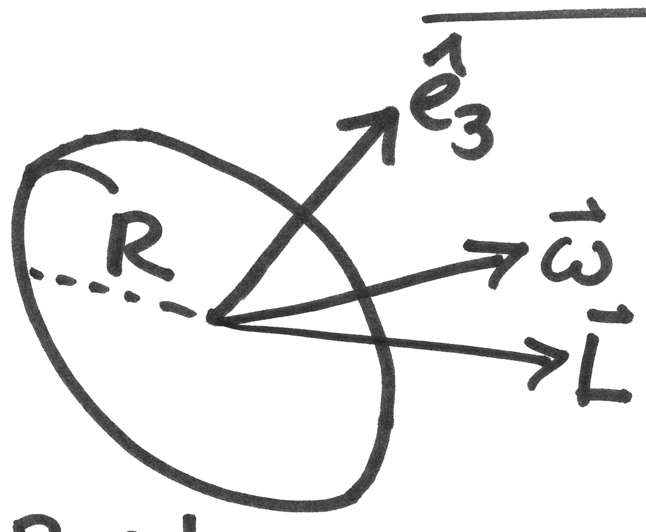
Free Precession of a Disk



$$\mathbb{I} = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{pmatrix}$$

$$\lambda_1 = \lambda_2 \simeq \tfrac{1}{4} M R^2$$

$$\lambda_3 \simeq \tfrac{1}{2} M R^2$$

$R \gg h$

$$\vec{L} = \begin{pmatrix} \lambda_1 \omega_1 \\ \lambda_1 \omega_2 \\ \lambda_3 \omega_3 \end{pmatrix} ; \left(\frac{d\vec{L}}{dt}\right)_{space} = 0, \text{ i.e. no torques!}$$

$$\left(\frac{d\vec{L}}{dt}\right)_{space} = \left(\frac{d\vec{L}}{dt}\right)_{body} + \vec{\omega} \times \vec{L} \Rightarrow$$

$$\dot{\omega}_1 = \left(\frac{\lambda_1 - \lambda_3}{\lambda_1}\right) \omega_3 \omega_2$$

$$\dot{\omega}_2 = -\left(\frac{\lambda_1 - \lambda_3}{\lambda_1}\right) \omega_3 \omega_1$$

$$\dot{\omega}_3 = 0$$

$$\Rightarrow \ddot{\omega}_1 = -\Omega_b^2 \omega_1$$

$$\ddot{\omega}_2 = -\Omega_b^2 \omega_2$$

$$\vec{\Omega}_b = \left(\frac{\lambda_3 - \lambda_1}{\lambda_1}\right) \omega_3 \hat{e}_3$$

$$\simeq \omega_3 \hat{e}_3 = \frac{L_3}{\lambda_3} \hat{e}_3$$

$$\vec{\Omega}_s = \vec{\Omega}_b + \vec{\omega}$$

$$= \frac{\vec{L}}{\lambda_1}$$

$$|\vec{\Omega}_s| \simeq \frac{\lambda_3}{\lambda_1} \omega_3$$

$$\simeq 2 \omega_3$$